NMR Quantum Information Processing: Successes and Challenges

Raymond Laflamme
Institute for Quantum Computing
laflamme@iqc.ca
www.iqc.ca

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Quantum Information Science in Canada
Goal: Investigate fundamental questions in Quantum Information

- Fellows
  - Prof R. Laflamme (Director and Ivey Fellow)
  - Prof G. Brassard (Montréal)
  - Prof R. Cleve (IQC and PI)
  - Prof A. Steinberg (Toronto)
  - Prof C. Crépeau (McGill)
  - Dr D. Gottesman (Perimeter)
  - Prof J. Watrous (IQC),

- Associates
  - Dr S. Aaronson (MIT),
  - Dr T. Jennewein (Vienna),
  - Prof B. Sanders (Calgary), Dr B. Terhal (IBM)

- Scholars
  - Dr A. Ambainis (IQC), Dr A. Blais (Sherbrooke), Dr J. Emerson (IQC), Dr P. Hayden (McGill), Dr P. Hoyer (Calgary), Dr D. Leung (IQC), Dr A. Lvovsky (Calgary), Dr M. Mosca (IQC and PI), Dr A. Nayak (IQC and PI), Dr M. Pioro-Ladrière (Sherbrooke), Dr R. Raussendorf (UBC), Dr A. Tapp (Montréal), Dr G. Weihs (IQC).
QuantumWorks

A research network with academic, industry and government partners to develop quantum technologies and their applications with headquarters at the University of Waterloo.

www.quantumworks.ca

Academic Partners:  Institute Partners:  Industry Partners:
IQC’s origin
Mission

Develop quantum information science and technology bringing together mathematicians, computer science, physicists, chemists and engineers.

Strategic Objectives

1. Establish an international centre of excellence doing research at the highest level
2. Become an international magnet for students
3. Become a source of information, analysis and commentary on the state of quantum information processing
Brief history

- Institute founded in 2002
- Mike and Ophelia Lazaridis donate $50M
- 2005: UW commits to 30 faculty positions
- 2006: $50M Grant from Gov’t of Ontario
- 2007: CFI/MRI approve $48M funding for fabrication facility
- 2009: $50M Grant from Gov’t of Canada
- Other grants (ORDCF, ARO, CRC, CFI, NSERC, MITACS) $15M
In 2008; 18 faculty, 21 pdfs, 64 students
Build to 30 faculty, 50 pdfs, 125 students
Building a World Class Facility

- Promote multi-disciplinary research
- 100,000 sq ft
- $80m
- next to 20,000 sq ft World Class Fab+Metrology

IQC
Institute for Quantum Computing
Research
Quantum Information Processing

Computation

Communication

Algorithms and protocols
Building blocks
Integration

<table>
<thead>
<tr>
<th>Theory</th>
<th>Q Computing</th>
<th>Q Communication</th>
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<tbody>
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<td>✓</td>
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</tbody>
</table>

Institute for Quantum Computing
Quantum error correction has been one the major achievement in QIP. Assumptions of the theorem:

- Good quantum control
- Parallel operations
- Ability to extract entropy
- Knowledge of the noise
  - No lost of qubits
  - Independent or quasi independent errors
  - Depolarising model
  - Memory and gate errors
  - . . .
Liquid state NMR

Cory & Havel PNAS, 64, 1634, 1997
Gershenfeld & Chuang, Science 275, 350, 1997

Qubits/Control/Measurement/Noise

• Larmor frequency:
  \[ \omega_L = \mu_p B_0 \approx 500 \text{ MHz} \]

• Single bit gate: \( H = \vec{\mu} \cdot \vec{B} \)
  \[ \sim \omega_L Z^1 + \mu(B_x X + B_y Y) \]
  \[ \sim 1/\text{kHz} \sim \text{ms} \]

• Two bit gate \( \sim 10 \text{ ms} \): \( \vec{J}^1 \cdot \vec{J}^2 \)
  \[ H_{int} \sim J_{12} Z^1 Z^2 \]

• \( T_2 \sim 1\text{s} \); \( T_1 \sim 10\text{s} \)

Histidine

\[ = |001001101011010\rangle \]
Initial state in Liquid state NMR

We have highly mixed state at room temperature.

• Making a “pseudo pure” state (Cory et al. 1996, Gershenfeld et al. 1997)

\[ \rho = \frac{1}{Z} e^{-\beta H} \approx \frac{1}{Z} (\mathbb{1} - \beta H + \ldots) \rightarrow \frac{1}{Z} \left( \mathbb{1} - \frac{\beta \omega_n}{2^n} |\Psi\rangle \langle \Psi| \right) \text{ (not scalable)} \]

BUT: Schulman and Vazirani (STOC, 322, 1999): Algorithmic cooling (i.e. concentrate polarization of the qubits) is scalable.

• The power of one bit of quantum information


- Characterization of complex quantum dynamics with a scalable NMR information processor Ryan, C, PRL 95, 250502, 2005.
- Estimating Jones polynomials is a complete problem for one clean qubit Shor, P. et a., QIC 8, 681, 2008.
- ...
Quantum Control

Implementation of quantum error correction:
- **3 qubits or phase quantum error correcting code**

\[
\alpha|0\rangle + \beta|1\rangle
\]

- **5 qubits quantum error correcting code**

Implementation of the 5 bit code with the stabilizer \(Z^2Y^3Y^4X^5, Z^3Y^3X^4, Y^2Z^3Z^5\) and \(X^1Z^2X^3Z^4\), including decoding and error correction for a basis of 1 qubit errors.

Cory et al., PRL 81, 2152, 1998

Knill et al. PRL 86, 5811 (2001)
Controlling larger Hilbert space

Controlling a dozen quantum bits
- need to use methods to minimize errors, compilers, grape pulses, refocussing etc.

Quantum Circuit: single bit + cnot
Ideal pulse program: X/Y 90, Z + ZZ

Pulse analysis
ideal, z, zz, rest

- select for rf
- calibrate

Output pulse program
(in Bruker language)

Data
Analysis

Ryan et al., PRA78, 012328, 2008
Characterising noise in QIPs

Can do process tomography, give all noise parameters, but scale as $4^{2n}$, already for 3 qubits the tasks is daunting.

QFT Superoperator

Theoretical QFT Superoperator

Experimental QFT Superoperator

Cory et al. JCP121, 6114, 2004
Instead can we get a few parameters that are useful? Coarse
grain the parameters and ask questions relevant to quantum
error correction, porbability to get zero ($P_0$), one ($P_1$), two
errors,..., independently of the type ($X, Y, Z$) and which qubits
are affected.

- Average the noise by randomising each qubits over $SU(2)$
  using 2-design techniques
- Randomise qubits
- Use sampling of randomising operators Chernoff bound

Start with the state $|00...0\rangle$ and measure the numbers of bits
flips, and repeat, and estimate the $P_i$.

<table>
<thead>
<tr>
<th>#</th>
<th>Map Description</th>
<th>Number operators ($A_k$)</th>
<th>$k_n$</th>
<th>$p_0$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Engineered: p = 0, 1, 0,</td>
<td>$\frac{1}{2}{Z_i, Z_k}$</td>
<td>286</td>
<td>0.000</td>
<td>-0.001</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>2</td>
<td>Engineered: p = 0, 0, 1,</td>
<td>${Z_i, X_k}$</td>
<td>288</td>
<td>0.001</td>
<td>-0.002</td>
<td>-0.004</td>
<td>-0.001</td>
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<tr>
<td>3</td>
<td>Engineered: p = 1/4, 1/2, 1/4,</td>
<td>${exp[{Z_i + Z_k}]}$</td>
<td>288</td>
<td>0.254</td>
<td>-0.495</td>
<td>-0.250</td>
<td>-0.500</td>
</tr>
<tr>
<td>4</td>
<td>Engineered: p = 0, 1, 0,</td>
<td>$\frac{1}{2}{Z_i, Z_k, Z_i}$</td>
<td>432</td>
<td>0.01</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>5</td>
<td>Natural noise (a)</td>
<td>$\frac{1}{2}{Z_i, Z_k}$</td>
<td>432</td>
<td>0.41</td>
<td>-0.41</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>6</td>
<td>Natural noise (b)</td>
<td>unknown</td>
<td>432</td>
<td>0.41</td>
<td>-0.41</td>
<td>0.15</td>
<td>-0.00</td>
</tr>
</tbody>
</table>

BENCHMARKING

Estimate the accuracy of gates such that:

- Device independent
- Independent of state prep and measurement
- Characterised by one parameter (strength of the depolarised noise)

G gates - Clifford group generators, P gates - Pauli gates, R gate - recovery operation

<table>
<thead>
<tr>
<th></th>
<th>liquid-state NMR</th>
<th>ion traps</th>
<th>superconducting qubits</th>
</tr>
</thead>
<tbody>
<tr>
<td>single-qubit</td>
<td>$1.3 \pm 0.1 \times 10^{-4}$ [1]</td>
<td>$4.82 \pm 0.02 \times 10^{-3}$ [2]</td>
<td>$1.1 \pm 0.3 \times 10^{-2}$ [3]</td>
</tr>
<tr>
<td>multi-qubit</td>
<td>$4.7 \pm 0.3 \times 10^{-3}$ [1]</td>
<td>$99.3%$* [4]</td>
<td>$55%$* [5]</td>
</tr>
</tbody>
</table>

Solid state NMR

- Stronger couplings
- Higher polarization
- Lower decoherence
- Control between the processors’ qubits and the bath
Algorithmic cooling

L. J. Schulman, et al., PRL 94, 120501, 2005

Compressing the purity into a smaller number of qubits, thermalize the hot qubits with a bath using solid state NMR

Baugh et al. Nature 438, 470, 2005
Ryan et al. PRL 100, 140501, 2008
Solid state NMR and ESR

Going towards hybrid systems

- Mehring et al. 2002: coherent transfer between nuclei and electron spin in malonic acid and $^{15}N@C_60$

- Cory et al. universal control of the nuclei via anisotropic hyperfine interactions

- Morton et al. (Oxford) using $^{31}P$ in $^{28}Si$, tomography of states

- Lukin ...