Quantum Control of Qubits and Quantum Systems

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Thanks to:
D. Bacon, S. Habib, H. Haffner, D. Lidar, H. Mabuchi, M. Sarovar, V. Scarola, L. Viola, D. Wineland, H. Wiseman, K. Young

Quantum control

1950

Laser

1960

1970

Spectroscopy motivated NMR -- *open loop* control on ensembles

Continuous measurement theory (Barchielli, Davies…)

1980

Formal open loop control methods (Rabitz, Warren, Dahleh…)
Many applications of open loop control to chemical dynamics

Quantum filtering and feedback theory (Belavkin)

1990

Quantum feedback control theory (Wiseman, Milburn, Doherty…)

Optimal open loop quantum control theory (Khaneja, Brockett, Rabitz…)

2000

Pulse sequences for qubits - dynamical decoupling, GRAPE, …
Quantum feedback control experiments (Mabuchi, Orozco, Blatt, Schwab…)
...

Outline

- Why do we want quantum control? (who needs it…)
- Open versus closed loop control, quantum vs classical
- Open loop control:
  - controllability, pulse sequences, decoherence, applications
- Closed loop control:
  -measurement-feedback, coherent-feedback, applications
- Quantum control for quantum information

Quantum information and Control

- Fault tolerance typically requires error per gate of order $10^{-3} – 10^{-5}$
- Current state of the art: ion traps

  1-qubit gate fidelity:

  Raman gates ~ 99.5(2) Knill et al, 2008

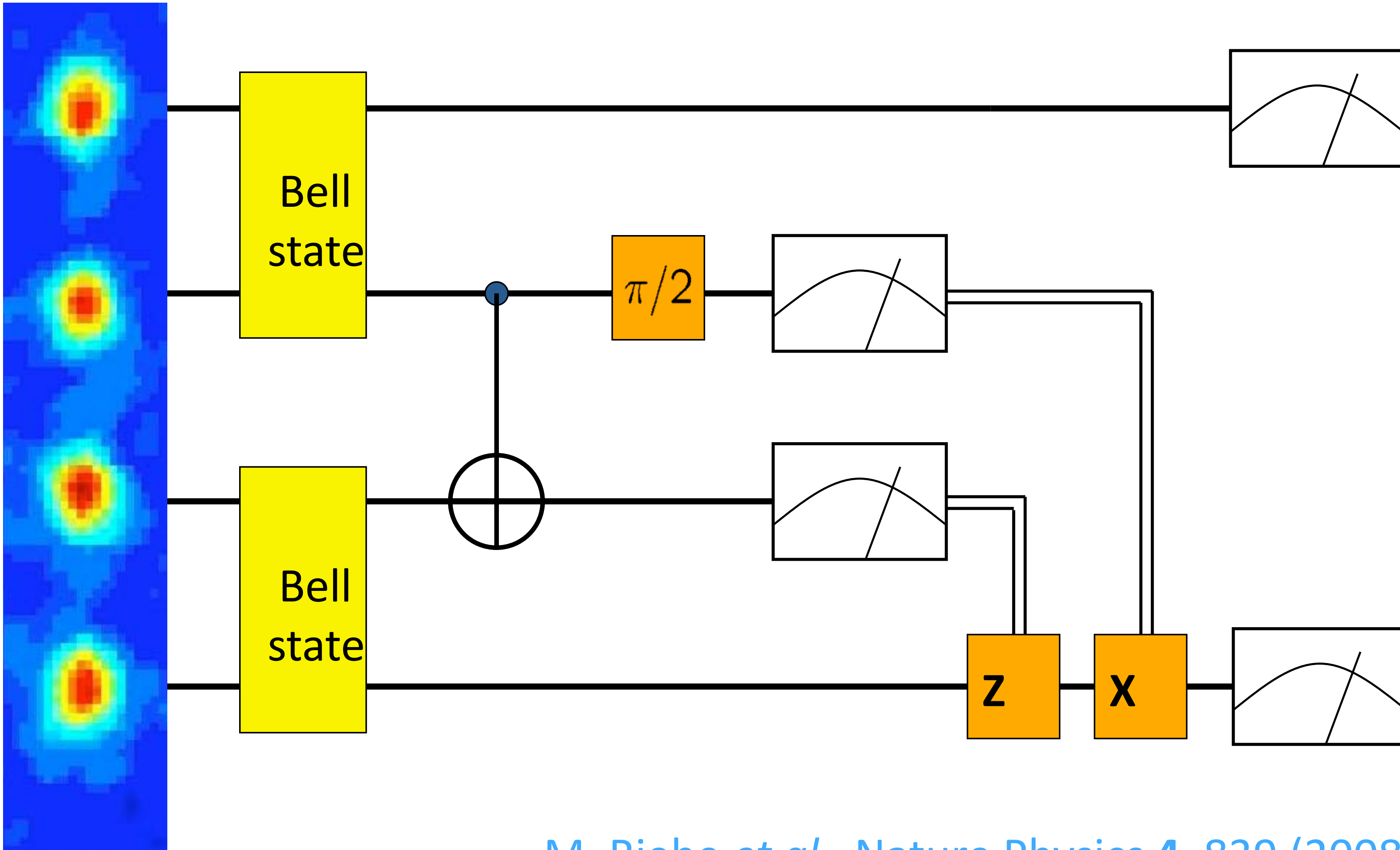
  RF gates ~ 99.9 Wineland et al, to be published

  2-qubit gate fidelity:

  process fidelity ~ 92.0 Riebe et al, 2006

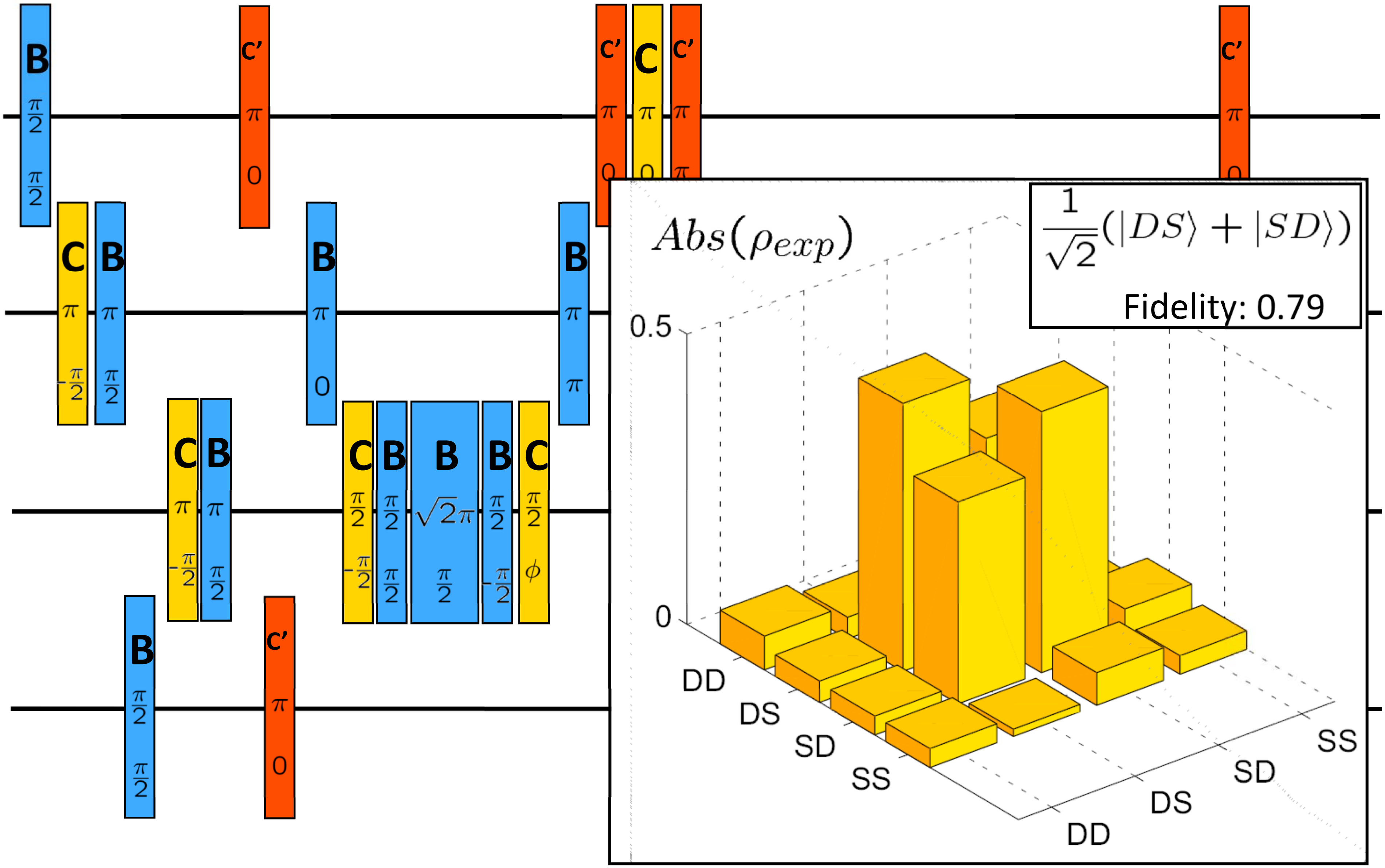
  Bell state fidelity ~ 99.3(1) Benhelm et al, 2008

Teleporting entanglement



M. Riebe *et al.*, Nature Physics **4**, 839 (2008)

Teleporting entanglement



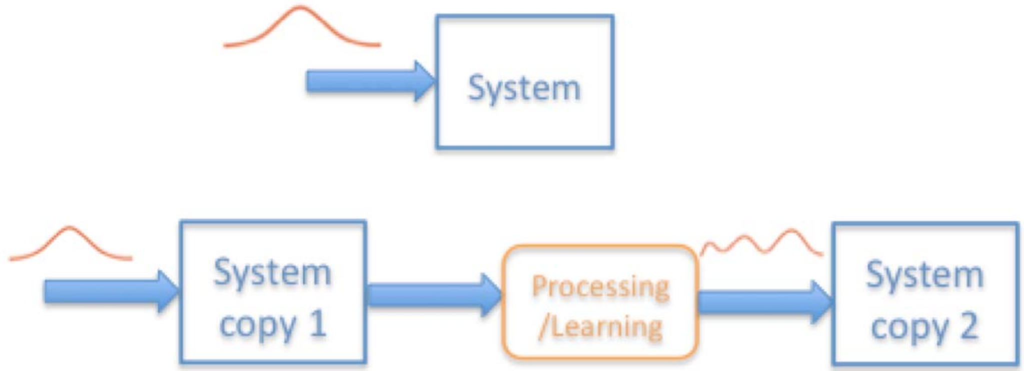
qubits clearly need control – are there other reasons why we should study *quantum* control?

One answer is that … integration of the theory of open quantum systems with estimation and control appears to provide an important new conceptual framework for the interpretation of quantum mechanics itself. By scrutinizing quantum mechanics as a theory for the design of devices and systems, as opposed to a theory for scientific explanation only, we gain new insight into obscure features of quantum theory such as complex probability amplitudes and 'collapse of the wavefunction'. In particular we are able to make more focused comparisons between classical and quantum probability theories.

A second compelling answer … is that various branches of research on nanotechnology are advancing to the point of investigating 'mesocopic' devices whose behavior remains quantum on timescales of functional relevance. … in order to fully exploit the powerful methodologies of control theory in the design and Implementation of advanced nanoscale technologies, control theory needs to be reconciled with quantum mechanics.

REF: H. Mabuchi, N. Khaneja. Int. J. Robust Nonlinear Control (2004)

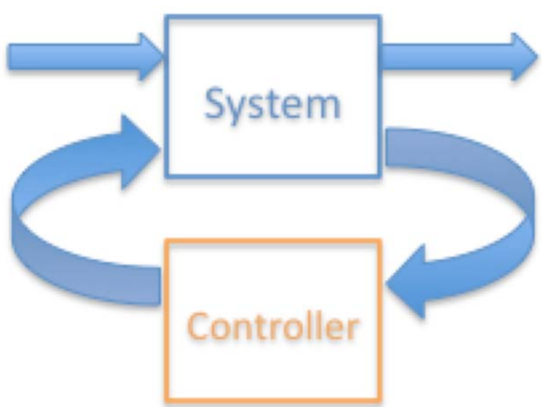
Open loop control:



-pulse train design, pulse shaping
-learning control

- system output not continuously monitored
- for slow systems/measurement
- similar principles for qu/class

Closed loop control:



- real-time continuous feedback control
- measurement conditioned evolution
  (cf. measurement-based QC)
- adaptive measurement: optimize info obtained from measurement
- coherent-feedback for dynamical control

- need fast measurements
- suited to stabilization, noise reduction
- differences from classical – i) quantum back action ii) new problem categories with coherent-feedback

Open Loop Control

- Antecedents in chemical dynamics, NMR
- Learning algorithms (sometimes also called ‘closed loop’)

- Here only consider open loop control of qubit systems!

Open Loop Control for qubits:

1. controllability, reachable sets, optimality:

$$\dot{U} = -i[H_d + \sum_{j=1}^{m} u_j H_j]U.$$

bilinear control problem

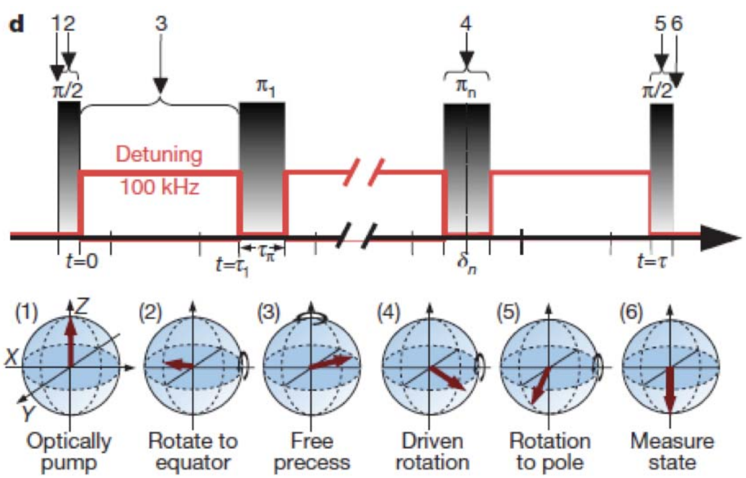
Lie algebraic/Geometric approach:
Khaneja, Brockett et al.
Zhang, Whaley et al.

- quantum gate construction from physical H (Whaley et al, '03, '06)
- optimal qu circuit construction (Zhang/Whaley, Nielsen, Markov, …'04)
- time optimal steering, e.g., coherence transfer (Khaneja et al, '02)

Open Loop Control for qubits:

2. Pulse sequences – analytic approaches (cf. CPMG)

Dynamical decoupling (DD) (Viola/Lloyd '99, Lidar et al, PDD, CDD,'03, '05)
Pulse interval optimization (beyond CPMG) (Uhrig UDD, '07, '09)
Dynamically corrected gates (DCG) (Khodjasteh/Viola '09}



- simple, robust, bath spectrum not required
- but - large number of pulses ($\geq 10^3$)
- many proposed applications of DD schemes to enhance quantum memory
- DCG protects against linear interactions during gates, uses bounded controls

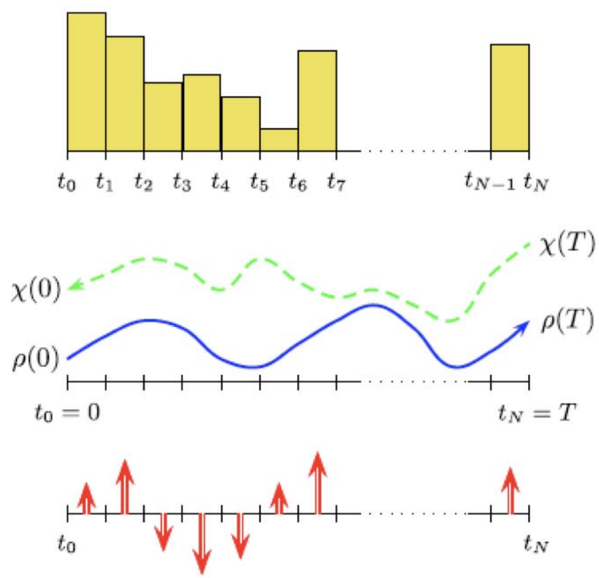
Some experimental implementations of DD schemes:
- 13C spin in adamantane (D. Suter) ~10-100 fold increase in T_2
- P donor qubit in Si (S. Lyon) ~15 fold increase in T_2
- trapped ions (Biercuk/Itano/Bollinger et al, '09) coherence time incr. by ~200

Open Loop Control for qubits:

3. Numerical optimal control with multiple pulses & shaping

- Optimal control theory application with numerical generation of pulse sequences of variable shape, length, amplitude,…
- versatile, realistic pulses, build in experimental constraints, bath spectrum..
- significantly shorter pulse sequences than DD methods

various optimization techniques, e.g., GRAPE, Khaneja, Glaser et al, 2005



Effect of pulse sequence monitored by forward/back evolution of Initial and final states. Formulate gradient from cost function to Evaluate corrections to pulse parameters.

Applications of numerical optimal control to qubits:

- Decoherence protection against Markovian noise, GRAPE

  (Whaley et al, ’06, ‘08)

- Fast control for coupled Josephson qubits, GRAPE

  (Sporl et al, ‘07)

- Decoherence protection against non-Markovian noise, by numerically minimizing overlap of bath response and pulse modulation spectra

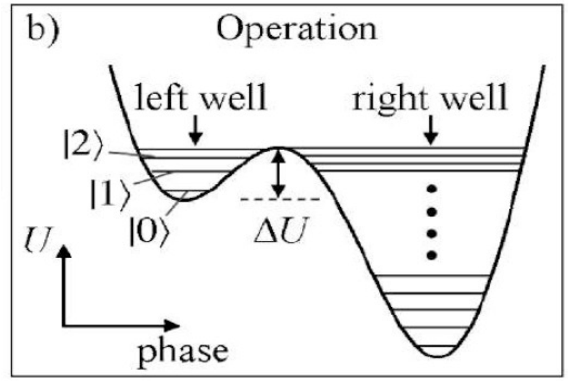
  (Gordon et al, ‘08)

- Leakage elimination in weakly non-linear qubits

  (Wilhelm et al, ‘08, ‘09)

High-fidelity gates in weakly nonlinear qubits

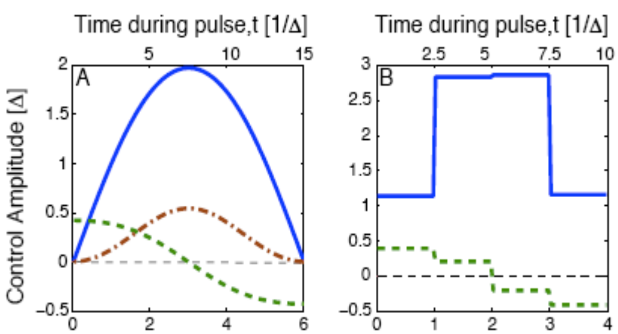
Qubits from multilevel systems:
- Superconducting: Transmon, Phase qubit
- Bus modes in Ion traps

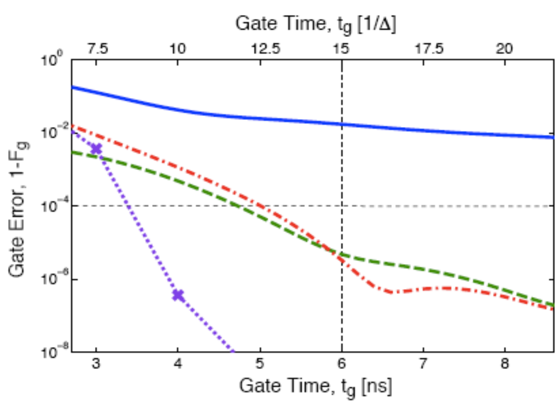


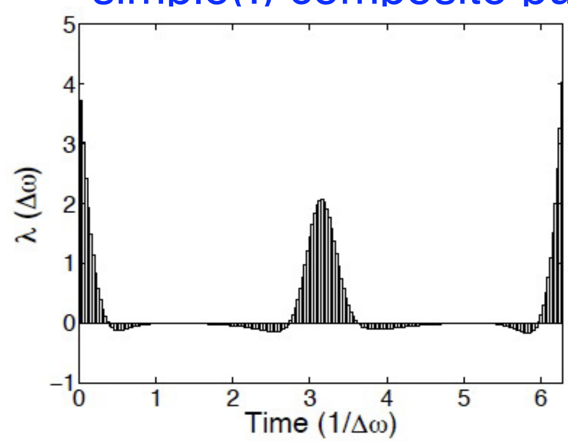
Leakage errors in fast, non-adiabatic gates

Two-quadrature control: DRAG
(Derivative removal by adiabatic gate)
even simpler(!) pulses…

Single-quadrature control:
simple(!) composite pulse







Blue: Gaussian
Green, Red:
Analytical/num.
Purple: GRAPE

P. Rebentrost and F.K. Wilhelm, PRB **79**, 060507(R), 2009; F. Motzoi, J. Gambetta, P. Rebentrost, and F.K. Wilhelm, arXiv: 0901.0534

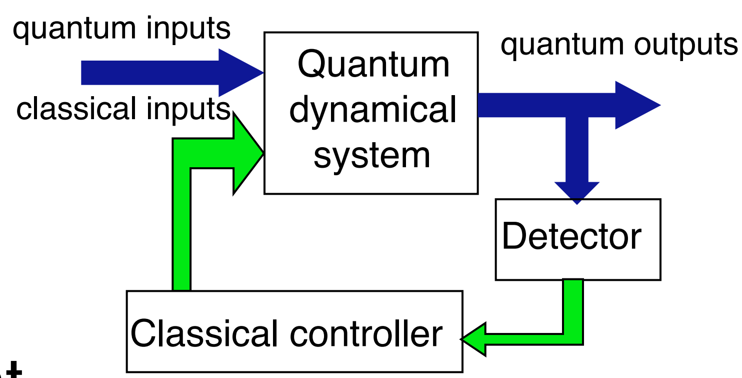
Closed loop (feedback) control

Classical	Quantum
Bandwidth of systems: Hz - GHz	Bandwidth of systems: MHz - THz --> faster controllers, faster processing
System state-space usually grows linearly	System state-space grows exponentially
Passive observation & noise reduction usually possible e.g. Kushner-Stratonovich equation describing the phase-space evolution of an ideally observed oscillator: $dP = \left[-\frac{p}{m}\partial_x - F(x,t)\partial_p\right] P dt + \sqrt{\gamma}(x - \langle x(t)\rangle)P dW,$	Measurement back-action unavoidable e.g. Quantum Kushner-Stratonovich equation (also: Belavkin equation, SME) for ideally observed oscillator: $d\rho = -\frac{i}{\hbar}[H,\rho]dt - (\gamma/8)[X,[X,\rho]]dt + \frac{\sqrt{\gamma}}{2}((X - \langle x(t)\rangle)\rho + \rho(X - \langle x(t)\rangle))dW,$ [K. Jacobs, arXiv: quant-ph/0605015]
No analogue	Coherent control possible – where the controller is a quantum entity Dynamical equations involve non-commuting stochastic variables (i.e. quantum stochastic differential equations)

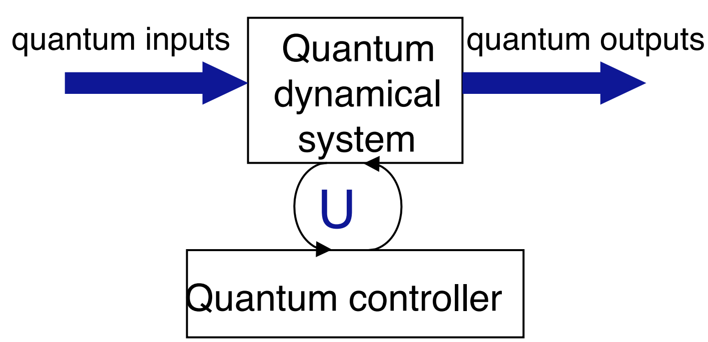
Quantum feedback control:

I. measurement-feedback (indirect)

adaptive measurement - feedback modifies measurement



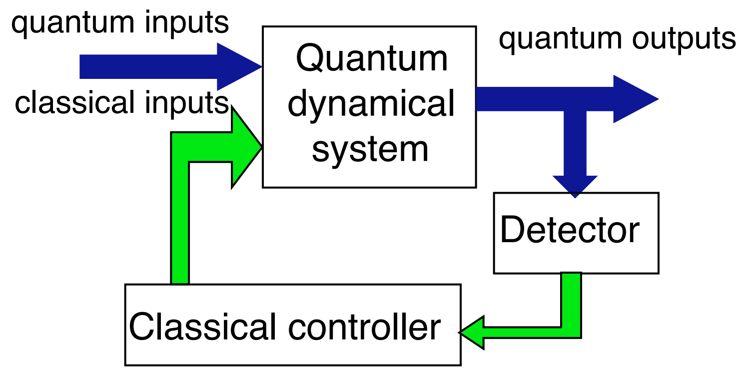
II. coherent-feedback (direct)



S. Habib, K. Jacobs, H. Mabuchi.
Los Alamos Science, **27**, 126 (2002)

Measurement-feedback control

relies on continuous monitoring, generating classical measurement record from which quantum state estimation can be made - classical control signals then fed back to system



Theory: many contributors
(Belavkin, Wiseman, Milburn, Doherty, Korotkov, Habib, Jacobs, van Handel, Bouten, Mabuchi, James…)

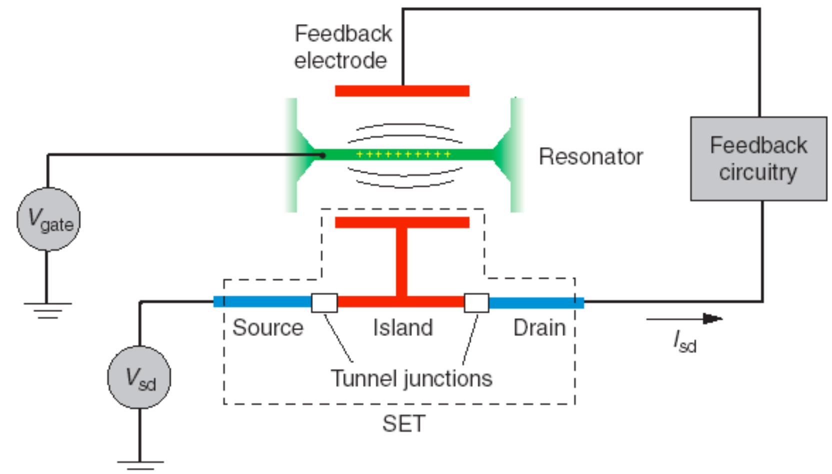
Theory protocols: cavity QED/circuit QED (Wiseman et al, '02, Steck et al, '04, '06/Sarovar et al, '04)
continuous quantum error correction (indirect) (Ahn et al, 2002, 2004)
atomic spin squeezing (Wiseman et al, 2002; Mabuchi et al, 2004)
cooling of trapped atoms (Doherty/Jacobs '99, Vuletic et al, 2004, Hope et al, '04,'07))
cooling of nanomechanical resonators (Jacobs/Habib et al, 2003)
squeezing of nanomechanical resonator (Korotkov et al, '05, Doherty/Milburn et al, '08)…

Experiments: generation of non-classical states of em field in cavity QED (Orozco et al, 2002)
adaptive measurement of optical phase (Mabuchi et al, 2002)
backaction cooling of nanomechanical resonator (Schwab et al, 2006)
manipulation of atomic motion optical lattice (Raithel et al, 2002)
feedback cooling of trapped ions (Blatt et al, 2006)
coherent state discrimination (Geremia et al, 2007)…

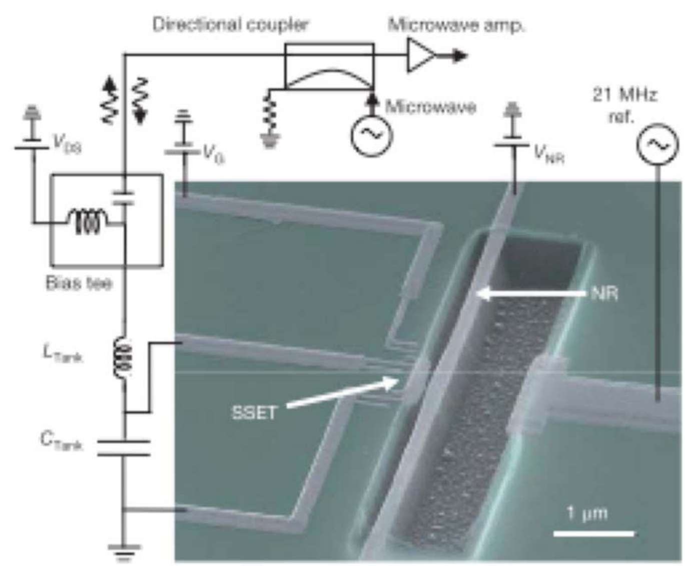
Nanomechanical resonator cooling by feedback:

Quantum Nanomechanics: Cool resonator mode via electrostatic coupling to the position. Need quantum limited measurements for strong cooling (e.g., to ground state).

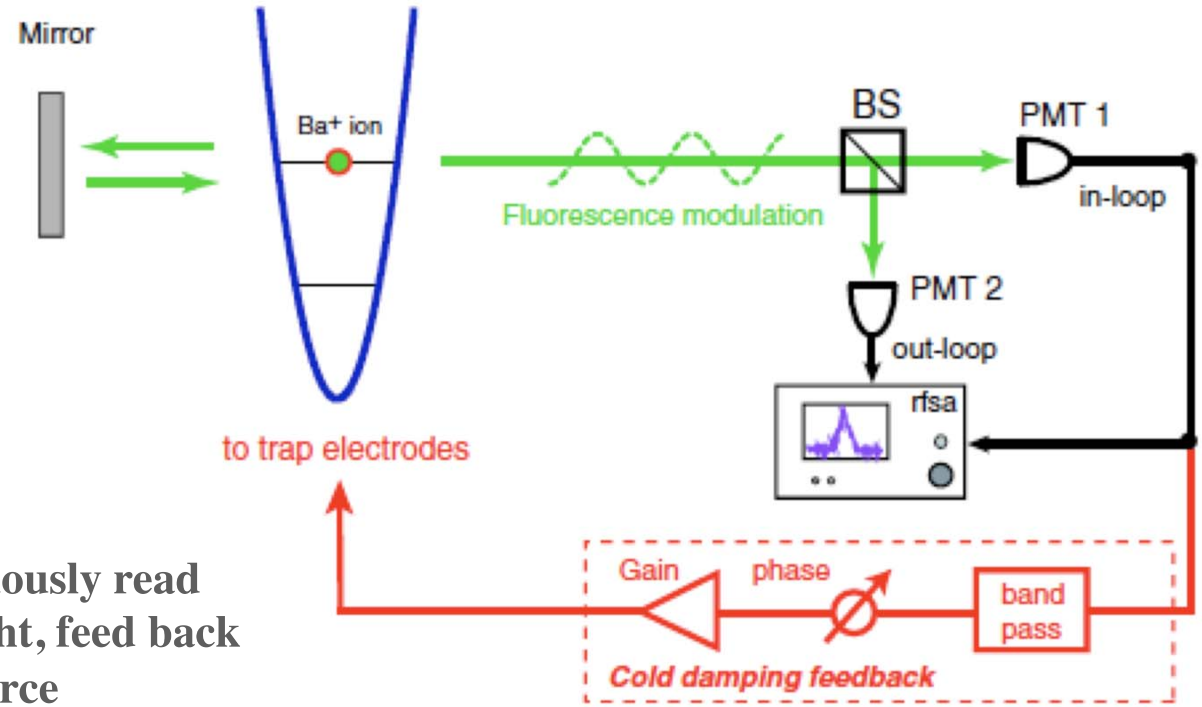
Theory: Hopkins et al, PRB (2003); Habib et al, LA Science (2002). Predict possible cooling to 0.35mK from 100mK.



Experiment: Schwab et al, Nature (2006). First step – use quantum backaction of measurement to damp oscillations, giving some cooling (passive). Observe cooling from 550mK to 300 mK.



Ion cooling by feedback:



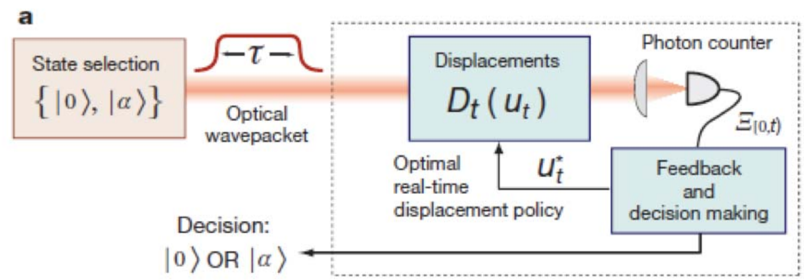
Single ion in Paul trap: continuously read position of ion from emitted light, feed back voltage providing a damping force proportional to instantaneous ion momentum to achieve feedback cooling.

Experiment: Blatt, Zoller, Rabl et al, PRL 2006. Electromechanical cooling below Doppler limit, achieving average occupation n=3.

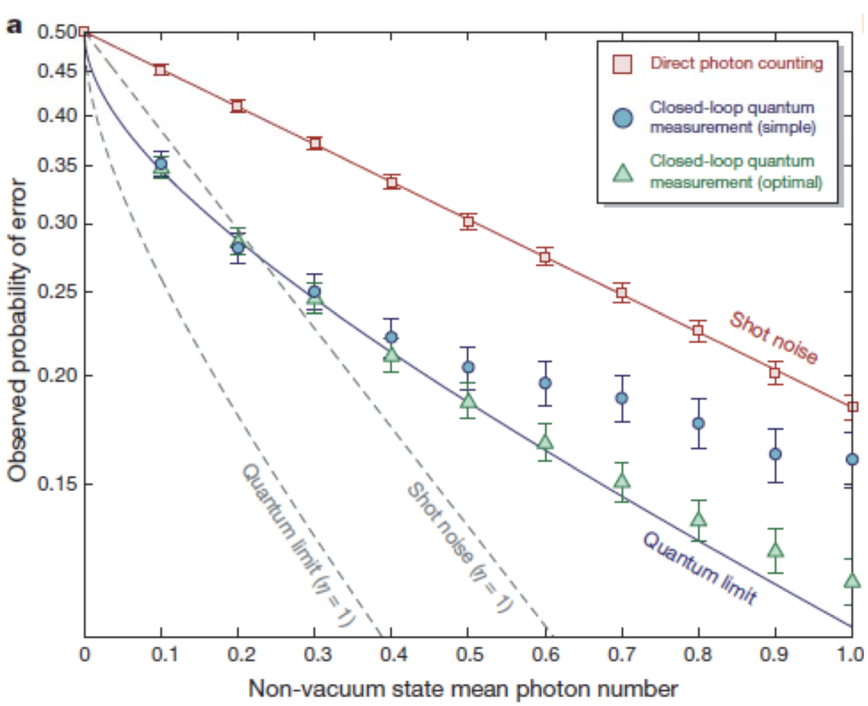
Optical Coherent state discrimination by feedback:

Feedback on optical field to emulate optimal (cat state) measurement:

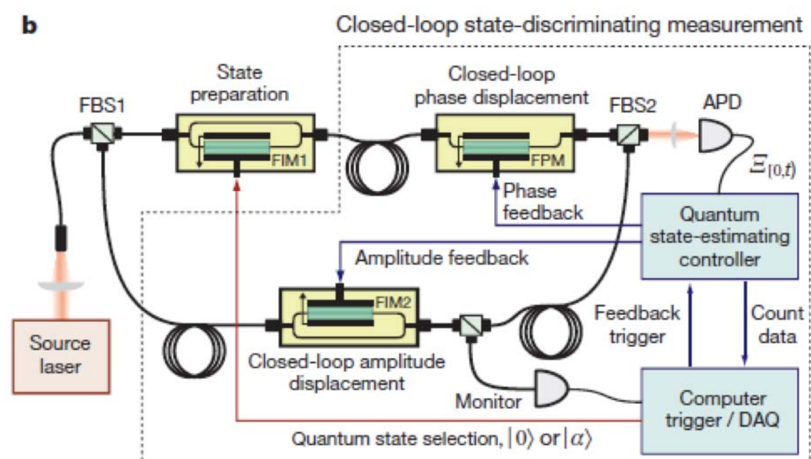
$$M = |m\rangle\langle m|$$
$$|m\rangle = c_0(\alpha)|0\rangle + c_\alpha(\alpha)|\alpha\rangle$$



Combine photon counting with feedback-mediated optical displacements applied during photon counting interval.



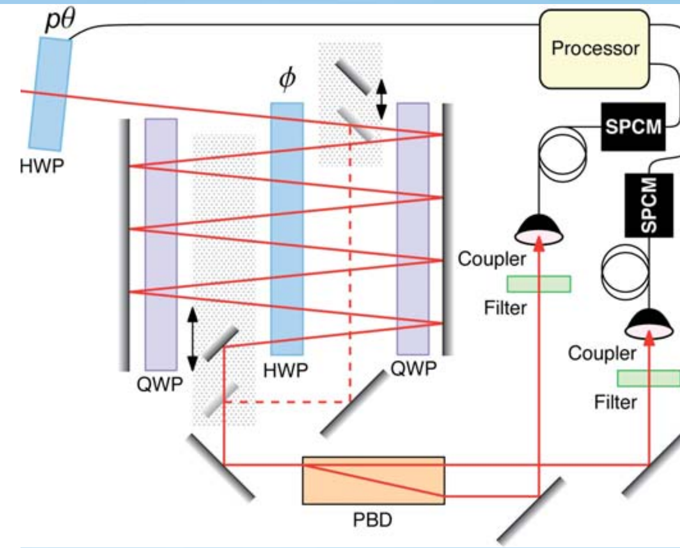
Experiment: **Geremia et al, Nature (2007)**



Adaptive measurements:

1. Heisenberg-limited Phase estimation: adaptive versus non-adaptive techniques.

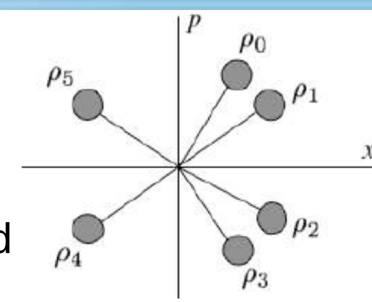
HL: $\delta\phi \rightarrow \pi/N$, with N=number of photon-passes through the unknown phase shift. Using multi-passes of single photons allows to reach much larger N. **Adaptive** technique gives HL scaling. **Non-adaptive** technique requires sophisticated resource partitioning.
Wiseman/Pryde et al, Nature 450, 393 (2007).

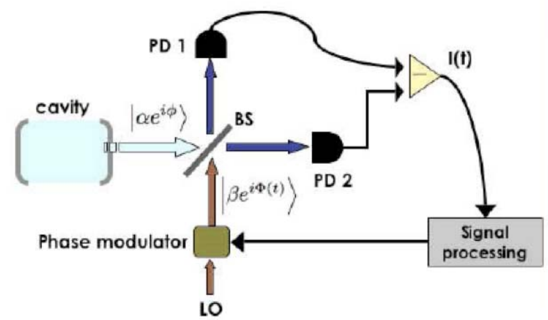


2. Qubit state discrimination: speed and accuracy of qubit measurement improved by adaptive measurement

Adaptive homodyne phase discrimination for N qubits coupled to harmonic modes. General strategy for large N, optimal for N=2. Adaptive strategy shows decreased S/N and time requirements for a given accuracy.

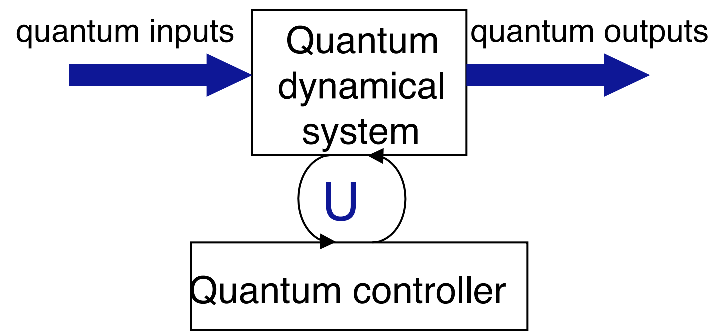
M. Sarovar and K. B. Whaley, PRA 76, 052316 (2007)





Coherent-feedback control

Direct quantum input path from controller back to system



Directly couple 'controllable' quantum system to plant

e.g., simple swap via discrete time control Cory, Lloyd et al., Phys. Rev. Lett., **85**, 3045 (2000)

$$|\psi\rangle_s \otimes |0\rangle_m \otimes |\phi\rangle_c$$
$$\rightarrow_U |\phi\rangle_s \otimes |0\rangle_m \otimes |\psi\rangle_c$$

e.g., continuous time QEC without measurement Sarovar, MilburnPRA **72**, 012306 (2004)

Linear coherent-feedback quantum control: 'unitarily' process output channel and redirect processed field back into system under control. Quantized fields implies non-commutative signals. Challenges for controller synthesis, optimization, extension to non-linear, non-Gaussian models.

Theory: James, Nurdin, Petersen IEEE-TAC **53**, 1787 (2008)

Experiment: coherent optical states Mabuchi, Phys. Rev. A **78**, 032323 (2008)

QEC by direct feedback

The bit flip code

Bit flip error: $|0\rangle \rightarrow |1\rangle \quad |1\rangle \rightarrow |0\rangle$

Encoding: $|0\rangle_L \equiv |000\rangle_P$

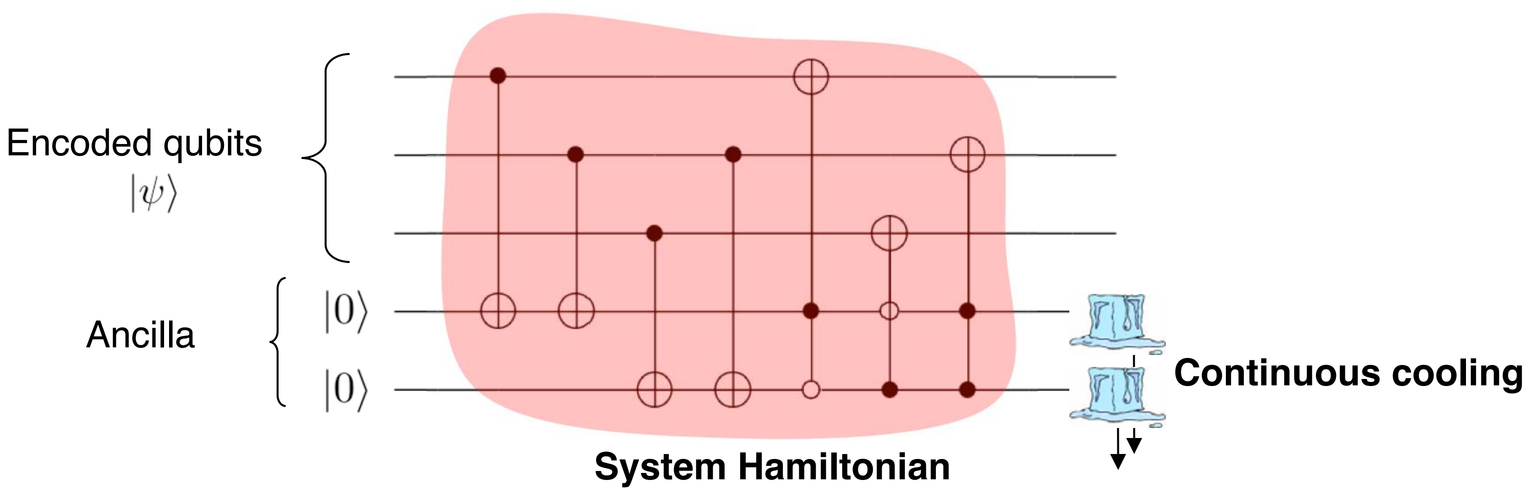
$|1\rangle_L \equiv |111\rangle_P$

$$|\psi\rangle = \alpha|0\rangle_L + \beta|1\rangle_L = \alpha|000\rangle + \beta|111\rangle$$

Stabilizer generators: ZZI, IZZ

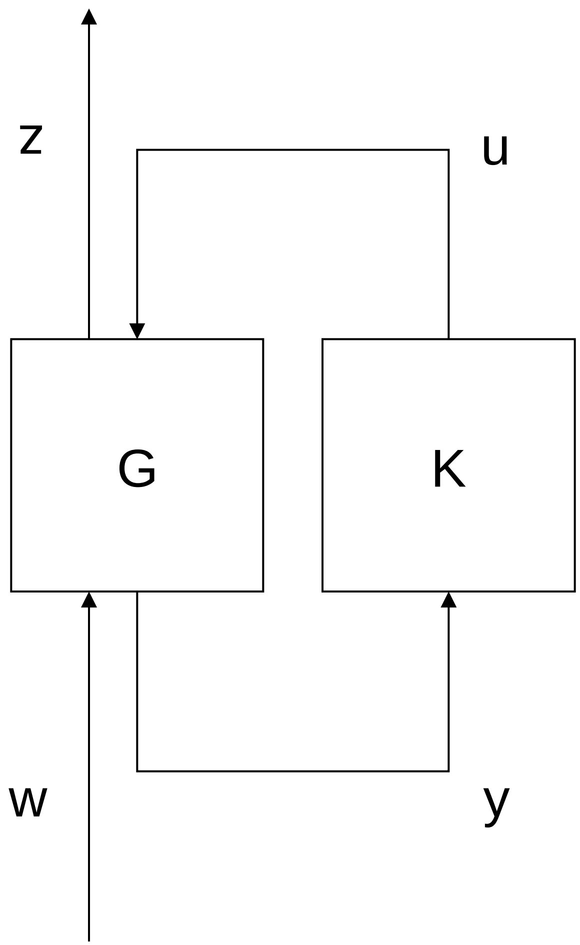
<ZZI>	<IZZ>	Error
+1	+1	None
-1	+1	On qubit 1
+1	-1	On qubit 3
-1	-1	On qubit 2

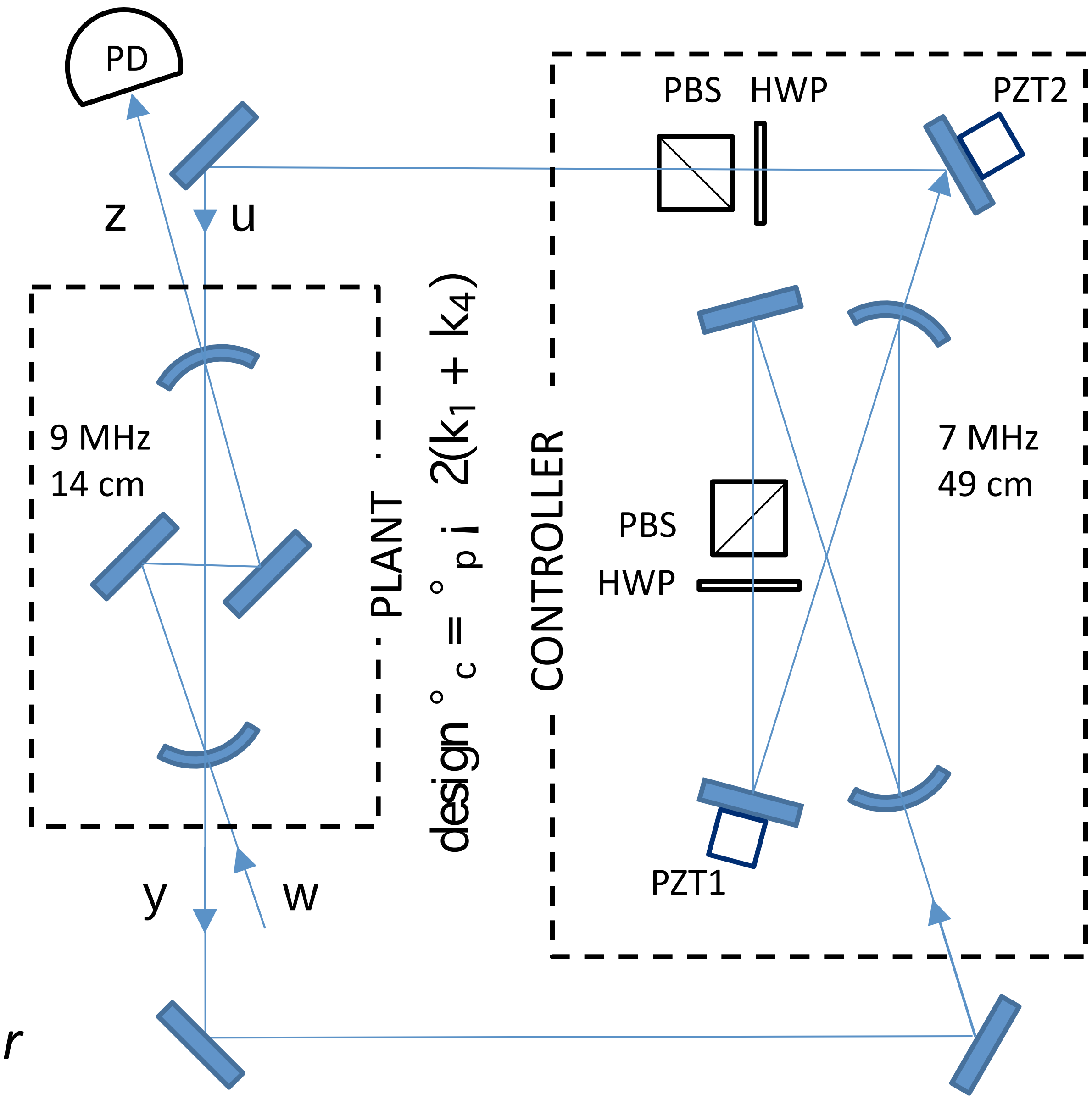
Sarovar/Milburn PRA **72**, 012306 (2004)



Disturbance rejection via coherent feedback

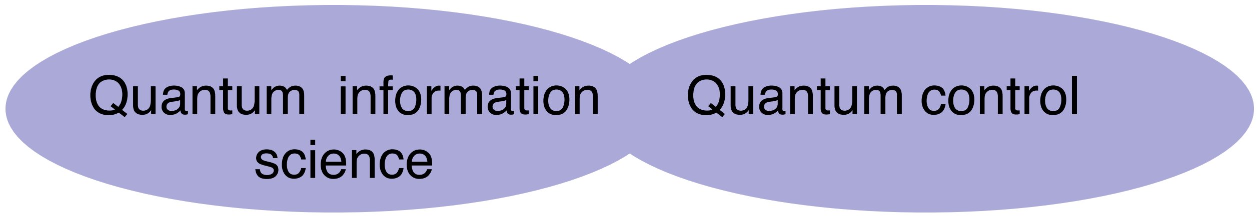
James, Nurdin and Petersen, IEEE-TAC **53**, 1787 (2008); Mabuchi, Phys. Rev. A **78**, 032323 (2008)





- *optical feedback*
- *dynamic compensator*

Quantum control and quantum information



- Quantum information sciences need quantum control for
  - Initialization / quantum state synthesis
  - Implementing computations/operations with qubits, especially for optimal implementation
  - Error correction, stabilization and noise reduction
  - State discrimination
  - Measurement …

- Quantum computing architectures are excellent experimental test-beds for quantum control of nanoscale technologies

- Control of quantum systems leads to formulation of coherent-feedback problems with no classical analog

- Understanding differences classical/quantum controllability -> fundamental physics need this for classical systems approaching quantum regime (nanoscale devices…)